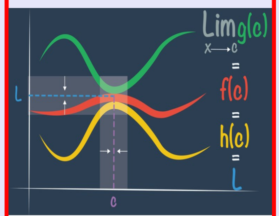
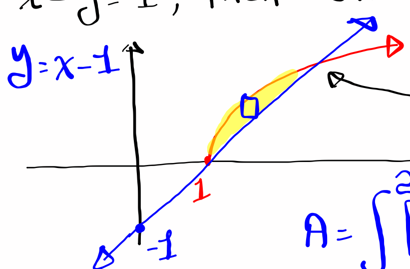
Calculus I

Lecture 56



Feb 19-8:47 AM

Sketch the region enclosed by $y=\sqrt{x-1}$ and $x-y=1$, then find its area.



$$\begin{cases} x-y=1 \\ y=\sqrt{x-1} \end{cases} \Rightarrow (2,1)$$

$$A=\int_1^2 \left[\sqrt{x-1}-(x-1)\right]dx$$

$u=x-1$

$du=dx$

$x=1 \to u=0$

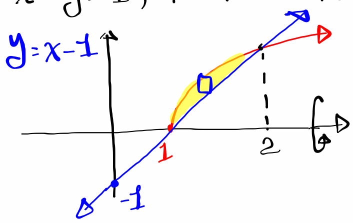
$x=2 \to u=1$

$$=\int_0^1 (\sqrt{u}-u)\,du$$

$$=\boxed{\quad}$$

May 28-8:45 AM

Sketch the region enclosed by $y=\sqrt{x-1}$ and $x-y=1$, then rotate about x-axis, find the Volume.



1) Ref. Rect. ⊥ A.O.R.

2) Region is not totally attached to A.O.R.

Washer Method

$$V=\int_1^2 \pi\left[\text{Top}^2-\text{Bottom}^2\right]dx=\int_1^2\pi\left[\left(\sqrt{x-1}\right)^2-(x-1)^2\right]dx$$

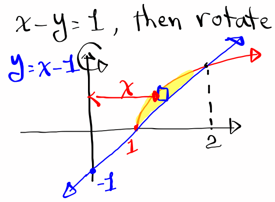
$u=x-1$

$$=\pi\int_1^2\left[(x-1)-(x-1)^2\right]dx$$

$$=\pi\int_0^1\left[u-u^2\right]du=\boxed{}$$

May 28-8:45 AM

Sketch the region enclosed by $y=\sqrt{x-1}$ and $x-y=1$, then rotate about y-axis, find the Volume.



Ref. Rect. is parallel to A.O.R.

Shell Method

$D=x$

$H=\text{Top}-\text{Bottom}$
$=\sqrt{x-1}-(x-1)$

$$V=\int_a^b 2\pi\, D\, H\, dx$$

$$=\int_1^2 2\pi\, x\left[\sqrt{x-1}-(x-1)\right]dx$$

$u=x-1$
$du=dx$
$x=u+1$

$$=\int_0^1 2\pi(u+1)\left(\sqrt{u}-u\right)du$$

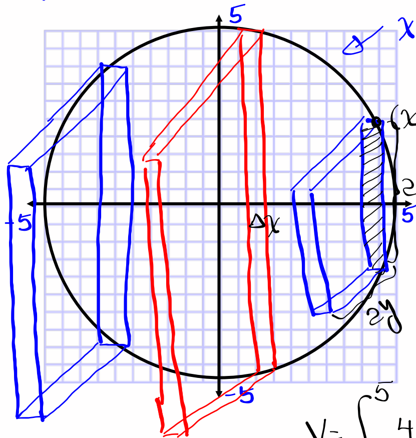
$$=2\pi\int_0^1\left[u^{\frac{3}{2}}-u^2+u^{1/2}-u\right]du$$

$$=2\pi\left[\frac{u^{5/2}}{5/2}-\frac{u^3}{3}+\frac{u^{3/2}}{3/2}-\frac{u^2}{2}\right]\Bigg|_0^1=\boxed{}$$

May 28-8:45 AM

The base of a Solid is a circle with radius 5.

$x^2+y^2=5^2 \rightarrow y^2=25-x^2$

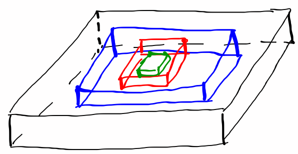


Parallel cross-Sections are squares and Perpendicular to the base.

Area of each square

$(2y)^2=4y^2$

$V=\int_{-5}^{5} 4y^2\,dx = 4\int_{-5}^{5} \underbrace{(25-x^2)}_{\text{even integrand}}\,dx$

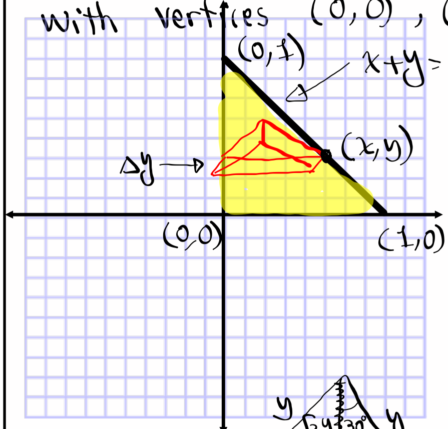


$=4\cdot 2\int_0^5 (25-x^2)\,dx$

$=8\left[25x-\frac{x^3}{3}\right]\Big|_0^5 = \boxed{}$

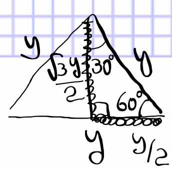
May 28-9:03 AM

The base of a Solid is a triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$.



Cross-Sections are $\perp$ to y-axis are equilateral triangles.

$\text{Area}=\frac{bh}{2}=\frac{y\cdot\frac{\sqrt{3}\,y}{2}}{2}$



$\text{Area}=\frac{\sqrt{3}}{4}y^2$

$V=\int_0^1 \frac{\sqrt{3}}{4}y^2\,dy=\frac{\sqrt{3}}{4}\cdot\frac{y^3}{3}\Big|_0^1=\boxed{}$

May 28-9:14 AM

Find f_{ave} for $f(x) = \sec^2 x \tan^3 x$ on $[0, \frac{\pi}{4}]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{ave} = \frac{1}{\frac{\pi}{4} - 0}\int_0^{\pi/4} \sec^2 x \tan^3 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = 0 \to u = 0$

$x = \frac{\pi}{4} \to u = 1$

$$= \frac{4}{\pi}\int_0^1 u^3 \, du$$

$$= \frac{4}{\pi} \cdot \frac{u^4}{4}\Big|_0^1 = \boxed{}$$

May 28-9:23 AM

Find f_{ave} for $f(x) = \frac{2x}{(1+x^2)^2}$ on $[0, 2]$.

$$f_{ave} = \frac{1}{2-0}\int_0^2 \frac{2x}{(1+x^2)^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$x = 0 \to u = 1$

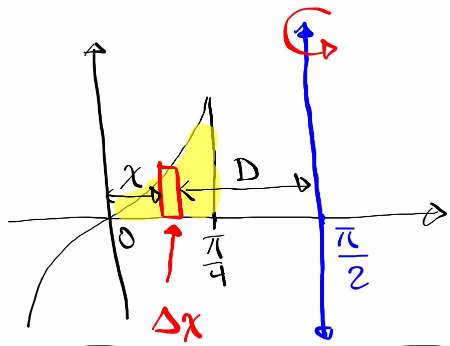
$x = 2 \to u = 5$

$$= \frac{1}{2}\int_1^5 \frac{1}{u^2}\,du$$

$$= \frac{1}{2}\int_1^5 u^{-2}\,du = \frac{1}{2} \cdot \frac{u^{-1}}{-1}\Big|_1^5 = \frac{-1}{2}\left[\frac{1}{u}\right]\Big|_1^5 = \boxed{}$$

May 28-9:28 AM

Set-up the integral to find the volume when rotating the region bounded by $y = \tan x$, $y = 0$, and $x = \frac{\pi}{4}$ by $x = \frac{\pi}{2}$.



Ref. Rect. is parallel to A.O.R.

Shell Method

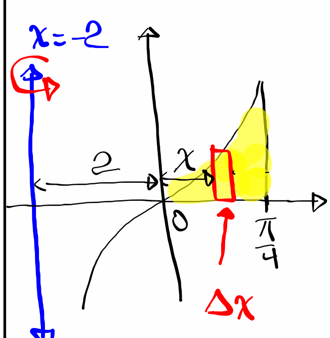
$D + x = \frac{\pi}{2}$ $\quad D = \frac{\pi}{2} - x$

$H = \text{Top} - \text{Bottom} = \tan x$

$$\boxed{V = \int_0^{\pi/4} 2\pi\left(\frac{\pi}{2} - x\right) \cdot \tan x \, dx}$$

May 28-9:34 AM

Set-up the integral to find the volume when rotating the region bounded by $y = \tan x$, $y = 0$, and $x = \frac{\pi}{4}$ by $x = -2$



Ref. Rect. is Parallel to A.O.R.

$D = x + 2$

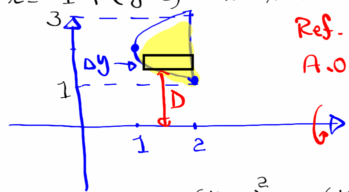
$H = \tan x$

Shell

$$V = \int_0^{\pi/4} 2\pi(x+2) \tan x \, dx$$

May 28-9:34 AM

$\boxed{\text{find}}$ the volume when the region bounded by $x = 1 + (y-2)^2$ and $x = 2$ is rotated by x-axis.



Ref. Rect. is parallel to A.O.R.

Shell Method

$D = y$

$H = \text{Right} - \text{Left}$

$x = 2 \rightarrow 2 = 1 + (y-2)^2 \rightarrow (y-2)^2 = 1 \rightarrow y - 2 = \pm 1$

$y = 2 \pm 1$

$y = 1,\ y = 3$

$$V = \int_1^3 2\pi\, y\left[2 - \left(1 + (y-2)^2\right)\right] dy$$

$u = y - 2 \rightarrow u + 2 = y$

$du = dy$

$y = 1 \rightarrow u = -1$

$y = 3 \rightarrow u = 1$

$$= 2\pi \int_1^3 y\left(1 - (y-2)^2\right) dy$$

$$= 2\pi \int_{-1}^{1} (u+2)(1 - u^2)\, du$$

$$= 2\pi \int_{-1}^{1} \left[u - u^3 + 2 - 2u^2\right] du$$

$$= 2\pi \left[\frac{u^2}{2} - \frac{u^4}{4} + 2u - \frac{2u^3}{3}\right]\Bigg|_{-1}^{1} = \boxed{}$$

May 28-9:43 AM